Culturally Adapted Mathematics Education with ActiveMath

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Abstract

Although it may seem that mathematics education material does not need any enculturation, the opposite is true. We report the results of a case study in several European countries and describe the different dimensions in which mathematics educational material has to be adapted to the cultural context of the learner. We describe the knowledge representation and mechanisms through which the user-adaptive learning platform ActiveMath realizes those adaptations to the language, countries, and communities of practice.

keywords: case study of intercultural mathematics, semantic standardization, enculturated web-based education

1 Introduction

At first sight mathematics does not appear to depend much on culture. As a result, most known applications of culture theories such as those in [6, 5] do not address mathematics and mathematics education. However, in section 5 we will show how some of those theoretical findings apply to mathematics and how they can provide hypotheses about these differences.

But even for mathematics-related publications cultural variety is mostly neglected. Take, e.g., the OMDoc 1.1 specification, which says ([9], page 27): "Note that Dublin Core also defines a Coverage element that specifies the place or time which the publication's contents addresses. This does not seem appropriate for the mathematical content of OMDoc, which is largely independent of time and geography."

However, national and regional history, dominating groups of mathematics in different parts of the world as well as internationally varying measure systems require a selection and presentation of mathematics in technology-enhanced learning of mathematics that is adapted to the culture of the learner. It is not only the language that requires enculturation but also the country context and in some cases the context of smaller communities of practice (CoP), e.g., the Bourbaki group of mathematicians or the readers of a particular book.

Communities of Practice have been introduced by Wenger in [22]. He acknowledges that (learning) practices are influenced by context(s) and that the internal dynamics of a CoP including its norms are determined by the various practices: "negotiation of meaning (i.e., understanding of mathematical notations and notions in our case)", "learning", as well as "community actions" (e.g., social rating), and "differentiation from other CoPs", where the last two may not be as important for the learning context discussed in this paper.

The inherent social interactions both produce and build upon CoP-specific cultural norms and specificities. For instance, any CoP produces abstractions, symbols, stories, terms, and concepts that reify something of its practice in a congealed form.

Communities defined by language or country can generally be seen as CoPs. In the following, we address as "CoP" only the somewhat smaller communities. However, the same principles and needs for adaptation apply for all CoPs. Anyway, it is more natural to ask the learner to which community he belongs in a more hierarchical way and to provide different levels of adaptation. For instance, to adapt to the language of a learner requires other mechanisms and level than to adapt to a specific notion common for one German Federal State only.

Kohlhase [8] reasons about some of the interactions in communities of mathematicians. Our research has empirically collected, addressed, and realized the following needed (macro-) adaptations as well as the architecture and knowledge representations needed for cultural adaptation. The actual adaptations we consider are

- language-adapted user interface
- CoP- and culture-adapted mathematical symbols and measures
- CoP- and culture-dependent input of formulæ
- CoP-dependent choices of learning objects

- CoP- and culture-adapted names of mathematical theorems, etc.
- culture-adapted sequencing of learning objects
- curriculum-dependent learning paths.

The article is structured as follows: first, it elicits the dimensions that need to be adapted in a mathematics education system such as ACTIVEMATH [15]. Because the basis of our cultural adaptation for education is an empirical one, we present the results of a case study which uncovered a number of representative cases, where enculturation of a European mathematics education system is necessary. This case study focused on the area of calculus (at the levels of high school and first year university). Then, it describes the knowledge representation and techniques with which ACTIVEMATH can adapt to the language, culture, and CoP of the student (and of authors). Finally, the article discusses related theoretical work and research in which different adaptation techniques have been tried.

2 Dimensions of Cultural Variability in Mathematics Material

Although the mathematical language has aspects of a *universal language*, variations occur that are bound to the language, culture of countries, or other communities of practice. In this section we present the aspects in which mathematical (learning) material can differ from one community to another. In the following, we address not only the actual variations in notation, notion, units, and sequencing but also provide examples on which type of community – defined by language, country, community of practice – they depend on.

Even cultural variations that seem purely linguistic in nature such as names and notions have a cultural and historical background (not easily separable). This is why just a translation of words does not work for such educational material, as we observed in our case study. Reasons for such differences can originate from a dominant group of mathematicians in history and cultural ties to that group in certain countries, regions, and groups. A similar development is observable today for Communities of Practice as well.

Note that cultural differences are often present in language-defined communities. One reason may be that in mathematics the history of schools, research, teacher education, popular books which influence the education culture go back to a language-defined culture that was present before today's countries existed. Moreover, as it can be observed, e.g., in physics, communities of practice that can be sociological structures orthogonal to country and language can also play a role in the emergence of cultural variability.

2.1 Notations

Mathematical notions and notations can differ:

- from language to language: e.g., the symbol for the binomial coefficient is C_n^k in French speaking countries and $(n\ k)$ in English speaking countries;
- from country to country: e.g., the Moroccan mathematical notation is written left-to-right (influenced by the French tradition) while the Machrek mathematical notation (used, e.g., in Egypt) is written right-to-left [11];
- from application to application: the symbol for the square root of -1 is i in most applications except in electrical engineering, where it is j (since I is the current there);
- from region to region: the German Federal States have different curricula/learning path and their textbooks differ, e.g., in notions and notations (see examples below) ;
- from CoP to CoP: e.g., N denotes the set of natural numbers which, in Germany, includes 0 for set-theory oriented users, whereas it does not for number-theory oriented users. A textbook/author may also indicate a CoP which is using the same textbook. Its author may introduce his own notations depending on usage, personal style, and typographical capabilities, e.g., "Einstein convention" pioneered in [4].

2.2 Units

Units such as meter, inches etc. are special symbols which occur in applications of mathematics. The most frequent differences originate from the imperial system of measures vs. the metric system. An educational system needs to respect these differences too when presenting examples or problems.

In addition to the imperial/metric differences, units can also be used in a country-specific way. For instance, in Belgium, land is measured in *are* while this same unit is deprecated by the originators of the metric system, the Bureau des Poids et Mesures [18].

Similarly, application domains use different units to measure. For instance, in physics the *J*oule is used for energy while in chemistry and *C*alories measure energy in dietetics.

2.3 Names and Notions

The names of theorems can vary depending on the language, country, or CoP as we shall see in section 3. Similarly, the notion, i.e., vocabulary for the mathematical concepts, can vary. For instance, the notion *permutation* is used in French to denote the bijections from a finite set to itself whereas it also denotes the *count of permutations without repetitions* $(n)^r$ in English. In French, this count is called *arrangements* and is written A^k_n. Later we shall see that names of theorems can vary according to the cultural context as well.

2.4 Educational Differences

The educational practice of teaching mathematics exhibits large differences between cultures. The most obvious differences are caused by the curriculum standards which differ greatly from country to country or even from region to region. For instance, the notions of similarity and similar triangles occur in England at Key Stage 3 together with the notion of enlargement (and consequently with the intercepting lines theorem) whilst in France the intercepting lines theorem is taught without any reference to similarity of triangles which is taught about two years later in another mathematical context (see [3, sec 2.2].

A pure translation of notions such as *instant slope* (English) or *momentane Steigung* (German) to the French *pente de la tangente* (slope of the tangent) is insufficient because the latter is based on geometric concepts which have other (learning) prerequisites than the prerequisites of instant slope. Hence, the context in which they are introduced differs.

3 Case Study on Cultural Variability

This case study about cultural variability in mathematics education has mainly been conducted in the European project ActiveMath-EU (in 2007-2008) that included Czech, Dutch, French, German, and Hungarian partners who contributed the results from their country and cultural point of view. This study was an exploratory study to observe how an average learning material would be translated from English and German to other languages with an eye on situations, in which a literal translation is inappropriate and needs cultural adaptation. There was no formal or very systematic procedure for the study. However, we are developing such a methodology currently which will also include the awareness for cultural variability.

During the process of “translating” learning objects and courses we expected and observed the need for adaptations of educational material for mathematics which go far beyond the obvious variations such as 1,5 vs. 1.5. This problem has then been addressed in research and technological solutions of the ActiveMath-EU and InterGeo projects. The empirical results are a rich source for enculturation of mathematics education systems and therefore, we address them in much detail here rather than just mentioning one or two cases.

In the ActiveMath-EU project cultural differences have not only been addressed for the general internationalization and the reuse of content on differentiation and its prerequisites (e.g., convergence of functions) but also for two other purposes, namely the bi-lingual teaching of mathematics (Hungarian and German in Budapest) and in a course for didactical comparisons (in Prague).

Intercultural issues arose at all the levels we describe in section 2. In the following, we summarize a number of examples, and in section 4 we describe the technological solutions we have developed for enculturating ACTIVEMATH and, thus, solving issues from the case study.

3.1 Symbolic Differences

The differences of logical notations such as $\forall$ vs. $\wedge$ and $\exists$ vs. $\vee$ are well known. In the studies we found quite a number of other symbolic differences, mostly depending on language.

One of them is the notation for coordinates of a point (4.5, 3) which is (4,5;3) in Hungarian, and (4,5 3) in German. Moreover, Figures 1 and 2 show what semantically equal notations for the Least Common Multiple (German: kleinstes gemeinsames Vielfaches) and a Binomial Coefficient (German: Binomialkoeffizient) look like when rendered for different languages by ACTIVEMATH.

en	de	fr	ru
lcm(5, 3)	kgV(5, 3)	ppmc(5, 3)	lcm(5, 3)

Figure 1: Least Common Multiple in several languages

en	de	fr	ru
C_m^n	$\binom{n}{m}$	$\binom{n}{m}$	C_n^m
C_m^n	$\binom{n}{m}$	$\binom{n}{m}$	C_n^m

Figure 2: Binomial Coefficient of two variables in different languages

3.2 Notions

Upper Bound. The notion of “upper bound” can be translated to German as “obere Schranke” and to French as “majorant”, whereas the “least upper bound” (German: “kleinste obere Schranke”) is “la borne supérieure” which is a literal translation of “upper bound” in French. In Czech, the word “majoranta” is used to denote the upper bound, but only for series. The upper bound for sequences is called “horni mess” and in this case, a human translator has to care about the context of the translation or the names have to be represented semantically and the actual presentation generated depending on the context reflected in the learning object’s metadata as described in section 4.4.2.

Nullfolge. A sequence converging to zero, e.g., a_n= (1/n), is called “Nullfolge” in German. In English the notion for this is “null sequence”. In Czech, it has no equivalent as a standalone concept. The literal translation of “Nullfolge” to Czech is defined and used only for the constant sequence of zeros, a_n= (0) which is a specific null sequence, in Czech. Therefore, the Czech translation is not using the literal translation but the semantic equivalent, again a description of the property “a sequence converging to zero”.

Convex, Strictly Convex. In Czech the notion “konvexní” and in German “konvex” is used for what is called weakly convex in English (sometimes also called just “convex” by some authors) and “ryze konvexní” (Czech) and “streng konvex” (German) for the English “convex” (sometimes also called “strictly convex” by some authors). Hence, the English version has always to refer semantically or syntactically to either “weakly convex”-“convex” or “convex”-“strictly convex” pairs. Cases like this illustrate differences between communities of practice rather then intercultural issues.

Strictly Increasing, Increasing, Strictly Decreasing, Decreasing. In Czech, for “strictly increasing” order the notion “rostoucí” is used, i.e., the “strictly” is omitted. The notion in German is “streng monoton steigend”. The Czech notion for (non-strictly) “increasing” order is “neklesající” , i.e., literally “non-decreasing”. The German notion is “monoton steigend”. Similarly, in Czech for “strictly decreasing” order the notion “klesající” is used (literal English translation: decreasing). The notion in German is “streng monoton fallend”. The Czech notion for “decreasing” order is “nerostoucí” (literally: non-increasing). The German notion is “monoton fallend”. So, the names have to be represented semantically and their presentations generated depending on the language.

In France, the order relation is assumed to be strict, otherwise it is not called an order at all. That is, in French context only the definition and semantics of “strict order” is available and can be used. If somebody wants to use the non-strict semantics, it presupposes rewriting of content.

Function. In the Hungarian tradition, a “function” includes the domain, the range and the rule for the mapping. The graph of the function is one of the representations of a function. This means, that a straight line or a parabola cannot be called a “function”, but just a “graph of the function”. This is handled more sloppily, e.g., in German.

Slope. Some definitions and names of concepts differ in incompatible ways between language-based communities. For instance, the same concept is called *instant slope* in English and *momentane Steigung* in German but is described in geometric terms in French, as the *pente de la tangente* (slope of the tangent). Similarly, in French curriculum, the naming of an *average slope* – “pente de la sécant” (slope of a secant) – is due to its definition via secants.

Depending on the context, the English term “slope” is translated into Czech by different words:

- Přírůstek grafu funce (in the context of functions)
- Stoupání (in the context of hills, roads etc.)
- Směrnice přímky (in the mathematical context of lines – a slope of a straight line)

In this case, one has to differentiate in which case the same concept has different names and in which case another concept is meant. A slope in the context of functions and hills (roads) describes the same concept, which is a generalization of the constant slope of a straight line.

Notion in the mathematics education differ in German Bundesländer, e.g.:

- Schaubild (in Baden-Würtemberg) vs. Funktionsgraph elsewhere
- (monoton) wachsend (in several states) vs . (monoton) steigend elsewhere
- Zentriwinkel (Thuringia) vs. Mittelpunktswinkel elsewhere
- Perihel (Thuringia) vs. Umfangswinkel elsewhere
- Basiswert for percentages (Nordrhein-Westphalen) vs. Grundwert elsewhere
- Mitternachtsformel (some including Nordrhein-Westphalen) vs. Lösungsformel der quadratischen Gleichung elsewhere.

3.3 Names of Theorems, Rules, etc.

Some examples for culture-dependent names have been given above in section 2.3. In addition, the names of theorems can differ in different languages and cultures. For instance, the Inscribed Angle Theorem is called “Theorem of Thales” in English and “Satz des Thales” in German, while “Theoreme de Thales” means an enlargement theorem in French.

As for region-related differences there are a number of examples for which schools in the German Federal States use different terminology, e.g., in Nordrhein-Westfalen the name “Mitternachtsformel” is used whereas in all other Federal States the same theorem is called “Lösungsformel für quadratische Gleichungen”.

Quotient Rule, Chain Rule. As opposed to German and English, in Czech, those names alone are insufficient. An additional information that these rules concern derivatives has to be included: Pravidlo pro derivaci podílu, Pravidlo pro derivaci složené Funkce. This is due to the fact that in Czech other rules which are not about derivation have similar names.

3.4 Curriculum Differences

Cultural differences due to curricula specific to countries or regions imply different sequencings of learning objects and notions.

Slope In French curricula, the average slope – “pente de la sécante” (literally: slope of a secant) – is introduced via secants, i.e., geometrically, rather than analytically. So, a common French course does not use an equivalent of the English definition of the “average slope” but other additional learning objects defining a secant. Consequently, when visualizing the concept in different cultures, different graphical representation might be used. In France, depicting the slope of a secant is used for illustrating the average slope, in Russia this is only used for application problems computing, e.g. an average velocity of a car.

Angular Sum. The proof of the formula for sum of angles is based on and presupposes the trigonometric and exponential form of complex numbers in Germany. Because complex numbers are not included in the basic curriculum in Hungary and the trigonometric functions and vectors are taught earlier than the exponential function (because of needs of Physics), the Hungarian curriculum has the concept “vector” as a prerequisite for the proof of the formula for sum of angles and the theorem is proven using vectors.

Educational Context of Country/Region. Depending on the country, more adaptation to the educational context may be needed. For instance, the calculus content of LEACTIVEMATH was considered appropriate for 11-13th grade at German gymnasium and Scottish first year university, but for Dutch secondary schools, the content was inappropriate because the educational context of Dutch secondary schools requires more interactivity and animation rather than just the formal mathematical concepts. This situation is similar to the “examples first” vs. “definitions first” sequencing strategies in different cultures. Similarly, the curricula of the different German Federal State differ. The different curricula can lead to different learning path because all prerequisites of a concept may have been taught to students in one state but not for students in another state. For instance, in year 8 of a common type of school (Realschule) the content to be taught in Baden-Würtenberg and Nordrhein-Westphalen is presented in the following table (in teaching sequence).

Baden-Würtenberg	Nordrhein-Westphalen

term rewriting	term rewriting
equations	data processing
perimeter and area	rectangles and polygons
linear functions	perimeter and area
systems of linear equations	percentage and interests
	linear functions

Moreover, at places the concepts taught for a specific topic vary depending on the state. For instance, for Gymnasium (secondary school) 8th grade in Rheinland-Pfalz students learn about probability calculus (among others) how to determine a probability through simulation, whereas in 8th grade Gymnasium in Nordrhein-Westphalen they learn to use Pascal's Triangle.

3.5 Tool Context

Computer algebra systems (CASs) differ in their semantics too: these computation tools do not only use different input syntax and rendering for the same function but in some cases different CASs denote semantically incompatible functions with the same name. An example is the usage of the inverse function of the tangens *tan*, written *arctan*, for which the domain differs in different CASs (Maple and Derive) and thus the denoted functions are different, see [2].

4 Adaptation Techniques in ACTIVEMATH

In ACTIVEMATH a user registers with an indication of his/her language, country, and learning context. As depicted in Figure 4, during learning, the user can dynamically change these properties using the *my-properties* menu. The given values are then introduced into the rather static part of the student model. The purpose of this part of the student model is the actual cultural adaptation. We consider it "static" since its values are rarely changed for a student as opposed to values for competencies and overall mastery of topics in a "dynamic" part of the student model.

According to the user's choice of country, language, and learning context ACTIVEMATH adapts its presentation of learning material. This adaptation includes a number of dimensions and, therefore, is technologically realized by a number of techniques. In addition to the technologies, a fundament for the cultural adaptation is the knowledge representation of learning objects in ACTIVEMATH that relies on an educational extension of the semantic representation OMDOC [9].

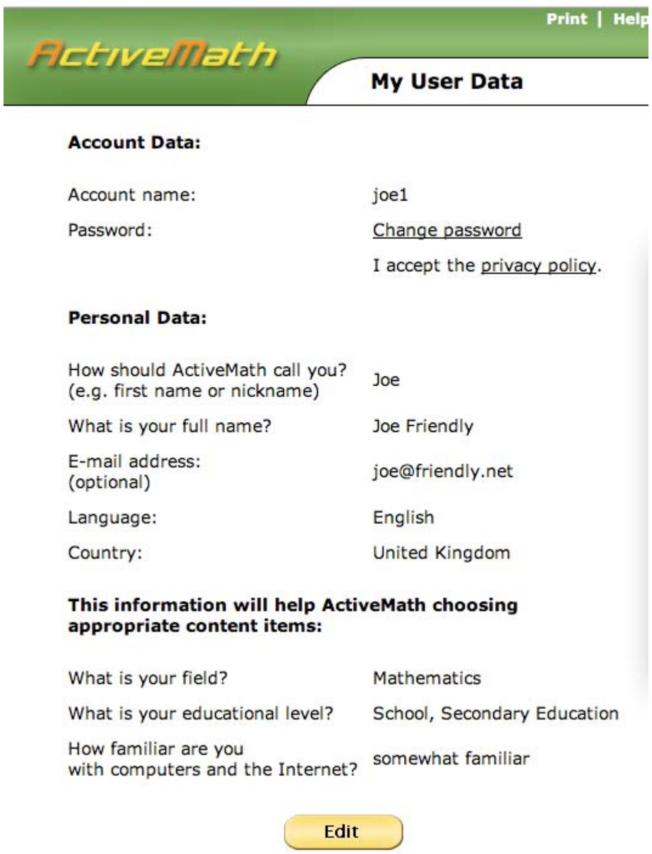

Figure 3: User profile as requested in the registration in ActiveMath

4.1 Architecture

This section provides an overview of ACTIVEMATH's architecture and its modules as far as they are important for the cultural adaptation.

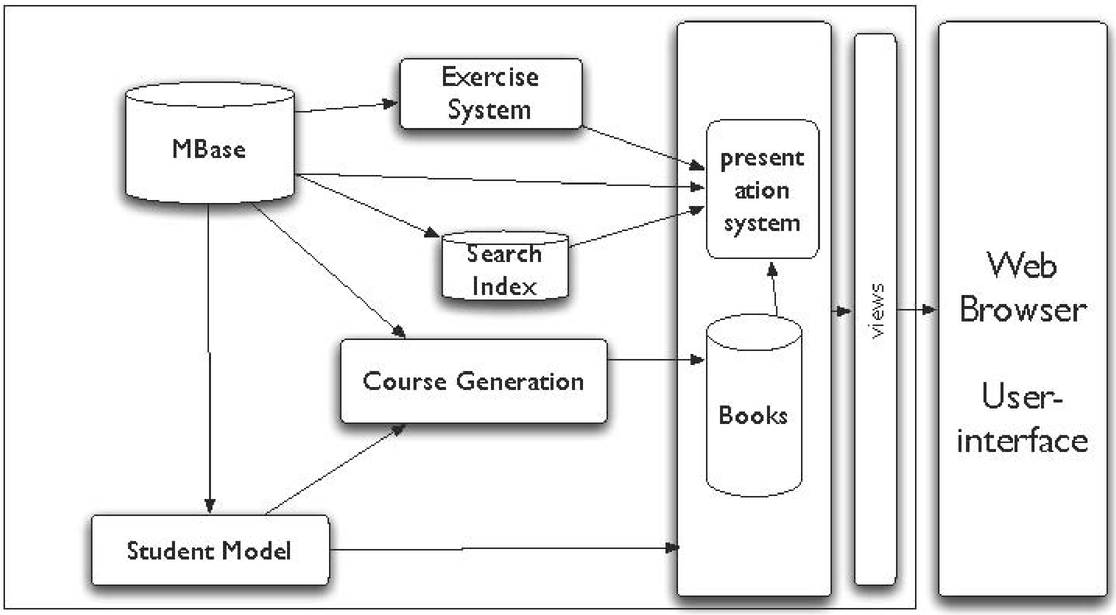

Figure 4: Architecture of the ACTIVEMATH system.

As described in more detail below, the following modules contribute to the enculturation of ACTIVEMATH. Their relationship is depicted in picture 4:

- semantic knowledge representation and its delivery through the MBASE content storage;
- the user interface;
- the presentation system;
- the course generator;
- the search module;
- the exercise system;
- the input editor.

4.2 Semantic Representation

ACTIVEMATH's knowledge representation of learning material is the semantic XML-representation for mathematical documents, OMDOC, with educational extensions. OMDOC itself is an extension of the semantic representation of OpenMath [1], an XML-representation of mathematical symbols (similar to MATHML-content).

In order to automatically adapt the learning material to the student's cultural context and to reuse the material, the structure of material is fine-grained and single learning objects are as small as possible and characterized (annotated) by culturally relevant metadata.

4.2.1 Content Items

The structure of material includes three levels:

- At the finest level, *content items* are defined and authored. They are the smallest elements with a mathematical and pedagogical significance. For example, a definition is an item, an exercise, a complete example.
- Content can be grouped in *theories* which form a coherent set of items that can be imported for further references. Content items are also grouped in *books* which provide a hierarchical organization of content items in chapters, sections, and page.
- At the coarsest level, content projects are gathered in *content collections* which group OMDOC items and web-resources. These collections are the unit of storage management and reuse which the content storage MBASE loads and indexes: it parses the OMDoc XML files and serves individual items as well as links from and to them.

4.2.2 Metadata Annotations

Each item is annotated with metadata and some metadata can be inherited from the theory level. Metadata in OMDOC include: general administrative metadata; mathematical metadata defining types of mathematical items ("definition", "theorem", etc.) and their relationships. These provide a basis for a domain ontology needed for course generation. In addition, pedagogical metadata serve the needs of course generation adapted to user's learning goals, personal preferences, and learning culture.

A widely accepted metadata standard which is included in any of today's educational metadata standards is Dublin Core.[1] Dublin Core metadata is used in OMDoc. It represents country/region-dependent priorities through the *coverage* metadata. For instance, a definition that is used within the region of Flanders is annotated with `<Coverage country="be" region="Flanders"/>`. ActiveMath uses the *coverage* metadata for its adaptive course generation, see section 4.6.

Language and *title* are other metadata that are used for enculturation in ActiveMath. The pedagogical metadata element *learningcontext* defines the education context, possible values of which can differ between countries.[2] EU partners in the LeActiveMath and ActiveMath-EU projects agreed upon a restricted set of values which are to be appropriate for their countries.

4.2.3 (Semantic) Textual Content

OMDoc encodes several types of mathematics learning objects such as definitions, theorems, axioms, examples, etc. These items are related via relations and constitute a rich domain ontology. Other domain and pedagogical and cultural metadata is attached to these items (see above). In ActiveMath the domain ontology is generated from the actual learning content and its metadata. The reason is that its educational content is fueled by a community of authors evolves over time. Of course, a community of authors (even with different cultural background) can agree upon and commit to a domain ontology before they start authoring content. This is, however, not the common situation for user-generated content on the Web.

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[1] `http://dublincore.org`

[2] The IEEE LOM standard had defined a set of values for this element which was later discarded, since there was no set of possible values approved that satisfies curricular wishes of all countries.

An item consist of text, hyperlinks, formatting elements such as lists, tables, as well as images and last but not least, semantically represented mathematical formulæ (see below). An item also contains a title for each language and textual fragments for each target language.

The multi-lingual nature of content items makes it possible

- assemble language-adapted material depending on the student's language;
- to keep the content's organization when a student switches languages, a feature that supports multi-lingual users.

4.2.4 Representation of Mathematical Formulæ and Symbols

The OPENMATH standard [1] defines an XML based encoding for mathematical objects based on elementary constructions such as binding and function application as well mathematical symbols whose semantics is described in content-dictionaries. OPENMATH defines an extensible set of symbols. The following is a definition of the binomial symbol (with two variables) that is located in the content-dictionary `combinat1`:

```
<OMOBJ>
   <OMA>
     <OMS cd="combinat1" name="binomial" />
     <OMV name="a" />
     <OMV name="b" />
   </OMA>
 </OMOBJ>
```

Symbols are special items in OMDOC and OPENMATH. OMDOC theories can contain `symbol` elements which themselves are explained in standard OPENMATH-content-dictionaries – the foundation of the OPENMATH standard. A symbol declaration is a semantic hook to which a symbol references points; it contains titles for each language and may be enriched with theorems and axioms within the same theory to indicate its properties.

Composed mathematical objects (expressions) written in OPENMATH can ensure interoperability: they refer to the standard semantics of their OPENMATH symbols and, thus, can be understood by any tool or process using/understanding the OPENMATH standard. Such tools include: input-editors to enter mathematica expression, rendering tools, evaluation/diagnosis tools such as computer algebra systems (CASs) or plotters, which are interfaced with a converter from and to OPENMATH that are called *phrasebooks*.

4.3 Internationalized Interface

Internationalization of the overall ACTIVEMATH is realized through the classical approach of phrases dictionary whereby developers insert phrase-catchers in the user interface which are replaced by the text of the appropriate language when rendered.

Currently, translating ACTIVEMATH requires the translation of about 1000 phrases of this dictionary, of the help collection, and of the mathematical formulæ rendering knowledge whose representation is explained below.

4.4 The ACTIVEMATH Rendering Architecture

The rendering process of ACTIVEMATH, which produces the browser material as delivered to the learner, converts the OPENMATH encoded formulæ to one of the presentation languages, HTML, MATHML, or TEX using a series XSLT files [7] which are fed by *notations*. This transformation is done in two stages [19] as illustrated in 5.

- The first stage uses only the language-relevant knowledge and applies XSLT to produce fragments. This is the place, where symbol presentations and notations are included.
- The second stage involves running the assembly of the content items into book pages and running other VELOCITY[3] code in the presentation: the latter can

[3] VELOCITY is framework that allows rendering parts of a Web page dynamically, similar to JSP. For instance, it can be used to insert the user's name in a welcome page.

employ all information relevant for cultural adaptation such as user profile or the domain of the material. This is the place where the actual rendering is finally adapted.

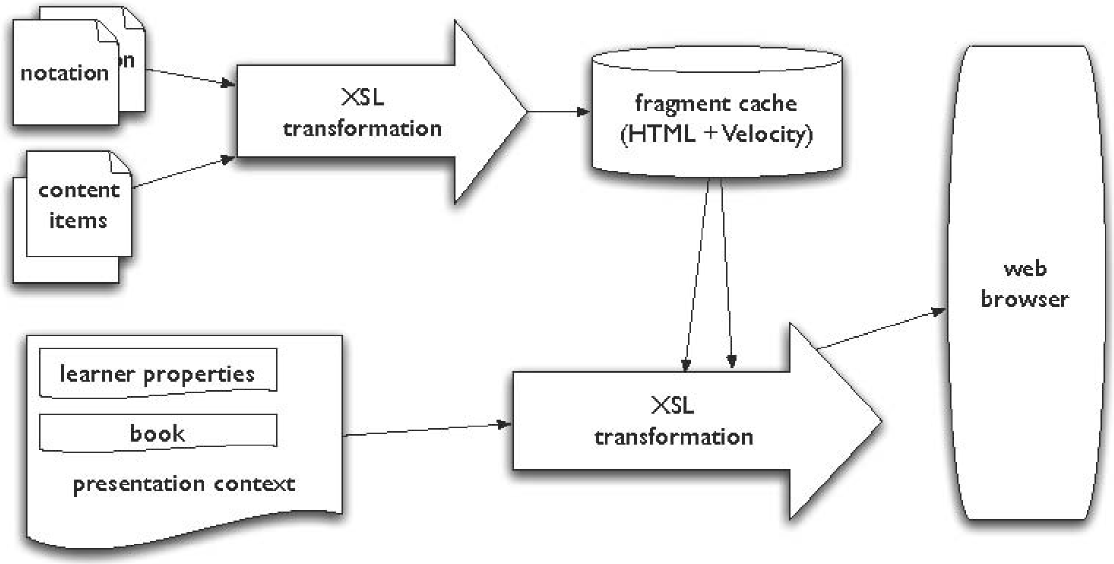

Figure 5: The steps of the ACTIVEMATH rendering process

4.4.1 Rendering Mathematical Notations

Rendering mathematical formulæ on the web is, in itself, an art because of the changing and fragile rendering techniques offered by web-browsers. MATHML-aware browsers offer the best rendering option and this is supported by ACTIVEMATH. Additional rendering options in ACTIVEMATH include HTML+CSS and TEX. HTML+CSS remains the default because it is better supported by browsers and, more importantly, interactive features can be added to the rendering to support the reading of the formulæ.

The rendering process of mathematical formulæ works mostly using XSLT stylesheets. For instance, for the notation for “binomial coefficient” (with variables a and b) as presented in Figure 2, a single representation can be rendered depending on the user’s language. The differences in notation that we have described in section 2 are realized by the rendering process based on the information of the symbol representation in Figure 6.

```
 <symbolpresentation id="combinat1binomial_59_54" for="binomial">
  <notation precedence="1000" language="en">
    <math>
      <msubsup>
        <mo>C</mo>
        <mrow><mi am:precedence="0">b</mi></mrow>
        <mrow><mi am:precedence="0">a</mi></mrow>
       </msubsup>
    </math>
  </notation>
  <notation precedence="1000" language="ru">
    <math>
      <msubsup>
        <mo>C</mo>
        <mrow><mi am:precedence="0">a</mi></mrow>
        <mrow><mi am:precedence="0">b</mi></mrow>
       </msubsup>
    </math>
  </notation>
  <notation precedence="1000" language="default">
    <math>
          <mfenced>
      <mfrac linethickness="0">
        <mrow><mi am:precedence="0">a</mi></mrow>
        <mrow><mi am:precedence="0">b</mi></mrow>
         </mfrac>
         </mfenced>
    </math>
  </notation>
  <OMOBJ>
    <OMA>
      <OMS cd="combinat1" name="binomial" />
      <OMV name="a" />
      <OMV name="b" />
    </OMA>
  </OMOBJ>
</symbolpresentation>
```

Figure 6: Representation of the binomial symbol for its language-dependent rendering

Essentially, the XSLT transformation matches the OPENMATH term and replaces it by the associated culture-dependent rendering and replaces variables by the rendering of their corresponding OPENMATH terms. All these features are collected and converted to an XSLT-template per symbol prototype. The template contains a condition on the language to output the right rendering for the given language. Then the template is converted to an XPATH expression.

Adaptation to further dimensions of variability is not dealt with XSLT but is coped with the notation declarations described above. Notable adaptation dimensions are a "book" collection and educational context. A specific notation may be rendered because it has a matching prototype and the material presented to the student belongs to the content collection for a specific "book" or the student has a specific educational context.

The XSLT stylesheet does not know of educational context or book-collection and thus outputs all possible cases, with a case split execution at the VELOCITY interpretation stage. The resulting enlargement of the presentation fragments, potentially exponential in terms of the formula depth is not significant, especially since the VELOCITY engine extremely efficient.

4.4.2 Enculturating Symbols and Names

In order to produce a knowledge representation of symbols as the one above, a symbol editor has been developed for ACTIVEMATH [13]. This authoring tool produces the notations (renderings) which are made of an OPENMATH prototype, the mathematical expression to be matched, and its associated rendering. The rendering itself is expressed in MATHML-presentation encoding.

The same rendering techniques can be used not only for modifying the presentation of symbols but also that of notions/names. Similar to symbols, the rendering facility fetches the title of the notion/name and places it during the preprocessing before the XSLT application.

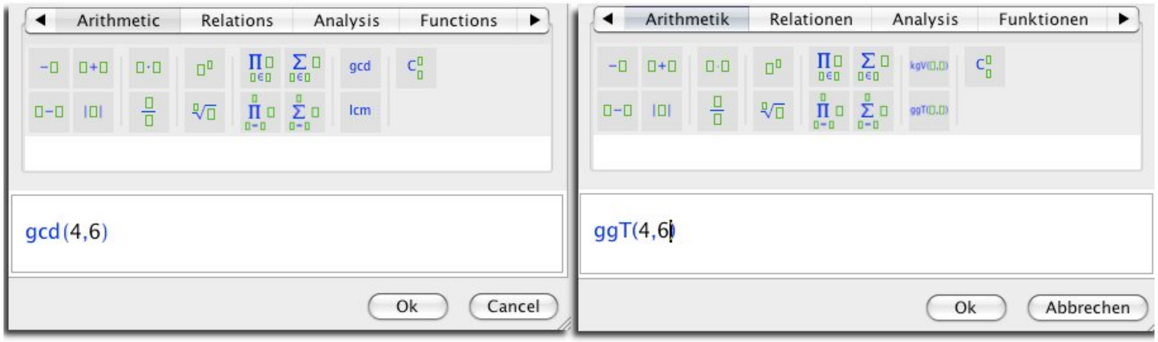

Figure 7: Customizing formula editor palettes for different languages

Apart from a culture-adapted presentation of mathematical notations ACTIVEMATH has culture-adapted formula input methods. Students as well as authors can use a palette-based input editor for mathematical expressions in ACTIVEMATH. At the moment, ACTIVEMATH's formula editor supports multiple notations for different countries. This support is static, i.e., there are static palettes for each country (see Figure 7).

4.6 Course Generator

Often, the culturally induced differences manifest in how learning objects are selected and sequenced. In particular, depending on the country or region, the curricula differ and the same mathematical concepts are described by relying on different definitions. Section 2.4 contrasted the example of *instant slope* in an English (and *momentane Steigung* in a German) curriculum with *pente de la tangente* in a French learning path. The French, geometry-based definition needs to be trained by examples different from those in the English and German counterparts.

These differences can be generated with ACTIVEMATH's course generator PAIGOS [20]. PAIGOS dynamically assembles the content items to courses, taking information about the learner into account, e.g., his language, country/region, learning context as well as his competencies and preferences. The assembly, called course generation, is performed according to formalized pedagogical knowledge. This knowledge encodes e.g., which definition, examples and exercises to present, in which order, and how the course is structured in sections and subsection. In total, PAIGOS's pedagogical knowledge encompasses about 360 "rules".

In previous work [20] we have shown that PAIGOS can implement various pedagogical strategies, such as those based on moderate constructivism as well as instructional design. PAIGOS's pedagogical knowledge is independent of the specific mathematical objects – however it uses information represented in the mathematical domain ontology. For instance, it uses the information represented in OMDOC about prerequisites of definitions, and does general pedagogical reasoning about the prerequisites (such as which ones to present). Thus, course generation in ACTIVEMATH takes information into account represented in the mathematical domain ontology and in a pedagogical ontology, but keeps these two ontologies separated. This makes reuse of pedagogical knowledge possible, such as applying the rules to not yet encoded mathematical areas. Neither of these ontologies is sufficient on its own for pedagogically-based reasoning: without information about mathematical concepts PAIGOS would not know which one to present; without pedagogical information only very limited courses could be generated.

Automatic course generation also takes country/regional differences into account. They affect the selection of appropriate examples, exercises and texts, but also the sequencing according to the prerequisite relationship. In the above example, a course generated for a French student presents the geometrical concept *tangente* first, followed by the notion of *pente de la tangente*. A course constructed according to the German way of sequencing the content shows the definition of (the graph of) a *function* first, and then the *momentane Steigung*. Thus, the courses generated for a French and a German student differ with respect to the presented definitions and prerequisites.

In order to explain how PAIGOS implements this cultural adaption, first we need to describe how learning objects are selected and assembled to a personalized course. During course generation, PAIGOS performs reasoning about learning objects. The underlying pedagogical and cultural knowledge is formulated with a vocabulary that is represented as an ontology of instructional objects (OIO) whose classes express different pedagogical purposes that learning objects can have [20]. For instance, during course generation PAIGOS might decide that at a particular place in the course (e. g., at instant slope) an exercise *f* or a concept should be inserted. This translates to a query for a learning object, which basically consist of a metadata constraint. In this example, the corresponding constraint (for a university student) is

```
(class exercise) (for instant_slope) (hasLearningContext university)
```

which contains the metadata *LearningContext* and *for*. PAIGOS sends these queries to the content storage, MBASE, collects the identifiers of those learning object that fulfill the constraint and return the resulting list to PAIGOS.

In order to realize the culturally implied differences in selection and sequencing, PAIGOS includes the location information of the student in the metadata constraint when collecting the prerequisites of a definition.

```
(:- (collectUnknownPrereq ?c ?result)
 ((learnerProperty hasEducationalLevel ?el)
  (learnerProperty hasLocale ?loc)
  (assign ?resources (call GetRelated (?c) -1
                        (((class Fundamental)
                         (relation isRequiredBy ?c)
                         (property hasLearningContext ?el)
                         (relation hasCoverage ?loc)))))
  (not (same ?resources nil))
  (assign ?sorted (call Sort ?resources
                       (((class Fundamental)
                         (relation isRequiredBy ?c)
                         (property hasLearningContext ?el))))))
  (removeKnownFundamentals ?reversedUnknown ?sorted)
  (assign ?result (call Reverse ?reversedUnknown))))
```

Figure 8: Inserting all unknown prerequisites

Figure 8 shows the formalized knowledge used to retrieve the prerequisites. Here, we focus on using information about the coverage of a learning object. In a first step, all fundamentals (the term in the OIO used to subsume definitions and theorems) that are required by the fundamental bound to `?c` (the question mark denotes variables) and whose learning context corresponds to the educational level of the learner are collected using the function *GetRelated* (lines 4–8) of Figure 8. This function retrieves all learning objects that are related via the given relation and that fulfill the given constraints. Here, one constraint includes the coverage of the learning objects. The coverage needs to correspond to the language and country preferences of the user (encoded in the value `hasLocale`). In case definitions or theorems were found that fulfill the conditions (line 9), they are sorted with respect to the prerequisite relationship `requires` (lines 10–13). Finally, those fundamentals that are known to the learner are removed using the axiom `removeKnownFundamentals` and the result is bound to the variable `?result`.

In ACTIVEMATH, each concept can have several definitions that differ in their Coverage metadata. If the Coverage metadata is not specified, the default ALL-countries is used. Upon a query, MBASE searches for the definition (more generally: learning object) whose Coverage corresponds to the user's country/region. If it exists, its identifier is returned. Otherwise, the identifier of a definition without coverage metadata is returned. If there is none, an empty list is returned. These search results are used by PAIGOS which inserts the definition in the course that is appropriate from the Coverage perspective whenever possible.

The here-described way of selecting prerequisites is one way of taking into account some of the cultural educational differences described in Section 2.4. A more advanced means is to modify the pedagogical knowledge itself, for instance to provide examples before introducing formal examples vs. first introducing the formal parts and only then the examples. The knowledge employed by PAIGOS is represented declaratively and thus is adaptable, but this currently requires help by an expert. Thus, until now, ACTIVEMATH ships with one large set of "rules", but for future versions a configuration tool is conceivable that allows a flexible selection of the knowledge used by PAIGOS.

4.7 Enculturated Search

ACTIVEMATH includes a search tool as many other systems do. Our experiences show that this search has to be enculturated too: not only the mathematical names and notions can differ between countries/languages but also curriculum standards may differ in their vocabulary used to describe mathematical concepts or competencies as indicated in [10] and [12]. For instance, where the French curriculum mentions concrete theorems about proportions, the English curriculum mentions only the recognition of a transformation.

In the INTERGEO project, these differences have been tamed so as to allow cross-curriculum search and annotation. The annotation user-interface remains in the language chosen by the annotator which, however, annotate topics, competencies, and educational levels which are part of an ontology; they are chosen following an auto-completion paradigm. For the search, the nodes of the ontology are used to expand basic text queries to queries for the corresponding ontology nodes and for nodes related to these. This offers the chance to find a related resource in a language that is different from the one of the original query language and resource.

The input of competencies and topics, in each users' language, is possible in two ways:

1. by pointing to the intended passage in the curriculum text of current use. Clicking on the intended line within a visually equivalent document is one possibility to input the (linked) topics and competencies
2. by searching for the competencies or topics typing a few words and choosing among an (auto-complete-like) list of suggestions.

Both input methods allow the user to "speak the ontology language". Most competence-topic nodes are presented by their titles and types. They are linked to a more elaborate web-page describing their attributes which is fundamental to verify the choices made.

4.8 Input Diagnosis

ACTIVEMATH’ default choice of units in exercises corresponds to the student’s country/language and on the exercise’s field.

Moreover, the exercise system diagnoses students’ input of mathematical expressions. In many cases, it determines whether the input is semantically (or numerically) equivalent to a given correct solution. Since the use of units/measures is culturally determined, the diagnosis has to convert units and their dependencies such as 1 meter = 100 centimeter.

For the diagnosis, units are handled as OPENMATH symbols and they can have definitions such as 1 inch = 2.53 cm and 1 meter = 100 centimeter. The exercise system queries a domain reasoner developed for units that finds out whether the student input is equivalent to the expected solution.

The numeric conversion between families of units is not difficult, e.g., meter to inch, feet, mile. However, the most appropriate has to be chosen (for which the resulting figures are most similar to the original pedagogical intention) and this is not (only) a question of numeric conversion. In such problematic cases the educational material has to be manually adapted.

The reason for this is pedagogical, and the selection of concrete numeric values in the problem statement of the exercise is not only bound to the proportions of the units in a unit system but also to the learning context and goal (e.g., learning to compute with integers or with real numbers). For instance, a problem statement of an exercise training proportions can be designed with imperial measures and based on the fact that 1 Yard consists of 3 Feet. In this context, it does not make much sense to numerically convert such a problem statement into the metric system using meters and centimeters.

5 Related Work

A recent attempt to make the content of the Assistment system for mathematics aware of culturally determined units and names is described in [21]. It describes how conditional variables are used for certain symbols and names. This equals a simple approach in QTI (Question & Test Interoperability) which is a standardized data format for online learning materials, mostly for quizzes and multiple choice exercises.[4] For instance, the variable "sports" can have the instances "baseball" (US), "Cricket" (India), soccer" (EU). The technique for these modifications of the Assistment system's content uses an introduction of variables into exercises. For instance, additional variables are introduced for names, e.g., American idol names. The variables" instantiations then depend on rules for values, e.g., sport depends on country (if US then Baseball) or currency depends on country (if US then Dollar). As compared with our approach this simple variable instantiation is restricted to (symbol) names and used depending on the student's country only. It yields a syntactic replacement which does not preserve the semantics of symbols as it is important for ACTIVEMATH.

Hofstede [6] defines culture as a dimensional concept and links differences in education to a culture's position on the dimensions. His dimensions include: (1) Power Distant Index (PDI - members accept and expect power to be distributed more or less unequally); (2) Individualism (IDV - degree to which individuals are integrated into groups); (3) Masculinity vs. femininity (MAS - distribution of roles between genders); (4) Uncertainty Avoidance Index (UAI - degree of tolerance for uncertainty and ambiguity); (5) Long-Term Orientation vs. short-term orientation (LTO - degrees of perseverance vs. tradition, social obligations, protecting one's face).

This theory can back hypotheses for some of the cultural variability discussed above, e.g., a low degree of PDI could be associated with teaching as a participatory process and require a discovery/exploration scenario in course generation while a high degree of PDI biases teaching towards a teacher-led more guided scenario. The MAS dimension may

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[4] http://www.imsproject.org/question/

not play such a big role because ACTIVEMATH adapts to the individual student anyway; a low UAI index may tend to a discovery scenario but only marginally because it seems that in most countries mathematics is taught without much ambiguity or uncertainty. A new direction that we are investigating is the inclusion of erroneous examples into learning material, for which students have to find errors and correct them [14]. Whether the effect of such learning objects depends on the student's culture is still unknown. LTO may require adapting tutorial dialogues and feedback to provide affective scaffolding with more/less autonomy and approval – two most relevant aspects that impact face [17]. This would yield a micro-adaptation (inside the exercises) which we did not address in this paper.

Hall's theory of encoding and decoding [5] focuses on negotiation of meaning which has also been a recent focus in math education research, where it is investigated from a cognitive rather than from a cultural point of view. Conclusions for computer-based learning include the usage of specific (e.g., visual) representations. In addition to the representations which support developmental change, we hypothesize that the provision of culturally adequate experiences – examples and counter-examples – will support the mental processes of assimilation and accommodation (see [16]) and, thus, influence the efficiency of a negotiation of meaning.[5]

The research on CoPs, e.g., in [22, 8] is interesting and tries to explain cultural variability based on groups of practice. It did not produce any technological solutions itself (as we did) and we think our empirical study contributed to better support the CoP ideas for mathematics learning.

6 Conclusions

Users/learners can adapt to almost everything but this adaptation consumes some of their cognitive resources which then cannot be activated for the actual learning process. Hence,

[5] Assimilation is the process by which new experiences are incorporated into existing mental structures. Accommodation is the process by which existing mental structures are modified to adapt to new experiences according to Jean Piaget.

culture-dependent adaptations can free students" cognitive resources for learning, i.e., for cognitive processes relevant for learning such as remembering and accommodation of old knowledge with new experience.

In our work, we took an empirical rather than a theoretical starting point. One obvious reason was that mathematics education did not get much attention in theoretical cultural studies previously. In this paper we therefore discussed a number of adaptation dimensions which are necessary in educational material for mathematics, which were observed in a case study and that are also presented.

We described how ACTIVEMATH performs many of these adaptations automatically. In particular, we show how the presentation and authoring of symbols, the presentation of units, names and notions are culturally adapted, and how the selection and sequencing of learning objects is enculturated. The technological basis for this has been implemented. More empirical work will be done in order to discover more instances for which enculturation is necessary. In a new project, we will also develop a systematic process for this discovery and provision, e.g., with additional gap detection and special focus on cultural variability.

Future Work ACTIVEMATH already has a mechanism of customizing notations. Only with respect to rendering of mathematical formulæ this task is fully achieved but the culture-dependent input syntax for mathematical formulæ through an input editor is not yet fully customizable – it is fixed per country. Such an input editor is needed for the input of mathematical expressions in a search tool or when solving an exercise,

Acknowledgement

This publication was supported by funding from the EU for the projects LeActiveMath (FP6-507826) and ActiveMath-EU (2006-4533/001-ELEB14) and Intergeo (ECP-2006-EDU-410016). We thank our partners who indicated the cultural differences in the first place. We also thank Jürgen Schmitt from Klett-Verlag for his input about variability in the regions of German Federal States. For additional aspects to be presented we like to thank the anonymous reviewers of this article.

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